Sample Question 1

Allotted time: 25 minutes (plus 5 minutes to submit)

Functions f, g, and h are twice-differentiable functions with $g(5) = h(5) = 1$. The line $y = 1 - \frac{5}{3}(x - 5)$ is tangent to both the graph of g at $x = 5$ and the graph of h at $x = 5$.

(a) Find $g'(5)$.

(b) Let b be the function given by $b(x) = 2x^2 g(x)$. Write an expression for $b'(x)$. Find $b'(5)$.

(c) Let w be the function given by $w(x) = \dfrac{3h(x) - x}{2x + 1}$. Write an expression for $w'(x)$. Find $w'(5)$.

(d) Let $M(x) = \dfrac{d}{dx}\left[\displaystyle\int_0^{2x} g(t)\,dt\right]$. Write an expression for $M'(x)$. Find $M'(2.5)$.

(e) Let $M(x) = \dfrac{d}{dx}\left[\displaystyle\int_0^{2x} g(t)\,dt\right]$. It is known that $c = 2.5$ satisfies the conclusion of the Mean Value Theorem applied to $M(x)$ on the interval $1 \leq x \leq 4$. Use $M'(2.5)$ to find $g(8) - g(2)$.

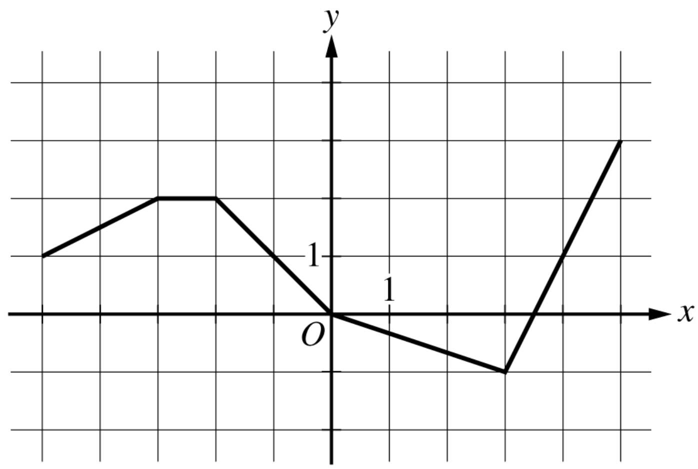
(f) The function g satisfies $g(x) = \dfrac{x + 5\cos\left(\frac{1}{5}\pi x\right)}{3 - \sqrt{f(x)}}$ for $x \neq 5$. It is known that $\lim_{x \to 5} g(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 5} g(x)$ to find $f(5)$ and $f'(5)$. Show the work that leads to your answers.

(g) It is known that $h(x) \leq g(x)$ for $4 < x < 6$. Let k be a function satisfying $h(x) \leq k(x) \leq g(x)$ for $4 < x < 6$. Is k continuous at $x = 5$? Justify your answer.

Sample Question 2

Allotted time: 15 minutes (plus 5 minutes to submit)

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

Let f be the function defined by $f(x) = \sin(\pi x) + \ln(2 - x)$.

Let g be a twice differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = 1$.

(b) Let k be the function defined by $k(x) = h(f(x) + 2)$. Find $k'(1)$.

(c) Evaluate $\int_{-5}^{-1} g'(x)dx$.

(d) Rewrite $\lim_{n\to\infty} \sum_{k=1}^{n} \left(h\left(-1 + \frac{5k}{n}\right)\right)\frac{5}{n}$ as a definite integral in terms of $h(x)$ with a lower bound of $x = -1$.

Evaluate the definite integral.

(e) What is the fewest number of horizontal tangents $g(x)$ has on the interval $-5 < x < 0$? Justify your answer.